Simple Closed Curves/The Isoperimetric Inequality/The 4 Vertex Thm-HW Problems

1. Does there exist a simple closed curve in $\mathbb{R}^2$with length equal to 6 enclosing an area of 3? Explain.

2. Does there exist a simple closed curve in $\mathbb{R}^2$ with length equal to 6 enclosing an area of 2? If so, find one (e.g. a rectangle).

3. Find the coordinates of all vertices of $y = e^x$ (parametrize this first).

4. Find the vertices of $y = \cos(x)$.

5. You can assume that there are only 2 vertices of the curve given by:

$\gamma(t) = \big((1 - 2\cos(t))\cos(t), (1 - 2\cos(t))\sin(t)\big)$.

(which occur at $t = 0$ and $t = \pi$). Explain why this doesn't contradict the four vertex theorem.