Smooth Surfaces- HW Problems

1. Consider the surface given by :

   $\vec{\Phi}(u,v) = (v\cos(u), v\sin(u), e^{-v})$; for $(u,v) \in [0,2\pi] \times [0,\infty)$.

Find the points where $\vec{\Phi}_u \times \vec{\Phi}_v = (0,0,0)$ (i.e., where $\vec{\Phi}$ is not regular).

2. Show that the elliptic paraboloid $z = x^2 + y^2$ is a smooth surface. Start by finding a suitable parametrization, $\vec{\Phi}(u,v)$, of this surface. Show that the parametrization, $\vec{\Phi}$, is smooth, one-to-one, onto, bicontinuous (i.e., both $\vec{\Phi}$ and $\vec{\Phi}^{-1}$ are continuous) and then that it's regular.

3. Consider the hyperbolic paraboloid defined by $S = \{(x,y,z) \in \mathbb{R}^3 | \, z = x^2 - y^2\}$. Check that $\vec{\Phi}(u,v) = (u+v,\ u-v,\ 4uv)$; $(u,v) \in \mathbb{R}^2$ makes this a smooth surface (i.e., show $\vec{\Phi}$ is smooth, one-to-one, onto, bicontinuous, and regular.)

4. Consider the surface given by $\vec{\Phi}: \mathbb{R}^2 \to \mathbb{R}^3$;

   $\vec{\Phi}(u,v) = (v\cos(u), v\sin(u), u)$, (this is called a helicoid). Show that the helicoid defined by $\vec{\Phi}$ is a smooth surface.

5. Let $\overrightarrow{\Phi}(u,v) = (uv,\ u(1-v), u^2(2v-1)),\ (u,v) \in \mathbb{R}^2$ be a parametrization of a surface S.

a. Show that points on S satisfy $z = x^2 - y^2$.

b. Either show that $\overrightarrow{\Phi}$ is one-to-one or show it's not by finding 2 points (u_1, v_1) and (u_2, v_2) such that $\overrightarrow{\Phi}(u_1, v_1) = \overrightarrow{\Phi}(u_2, v_2)$.

c. Either show that $\overrightarrow{\Phi}$ is onto $U = \{(x,y,z) \in \mathbb{R}^3 |\ z = x^2 - y^2\}$ or find a point in U not in the image of $\overrightarrow{\Phi}$.