The Derivative of a Function from R^n to R^m - HW Problems

1. Let $f: \mathbb{R}^2 \to \mathbb{R}$ by $(x, y) = \sqrt{|xy|}$. Show that f is not differentiable at $(0,0)$.

2. Let $g: \mathbb{R}^n \to \mathbb{R}$ where $|g(x)| \le |x|^2$. Prove that g is differentiable at $(0,0, \ldots ,0)$. Hint: Figure out what $Dg(0, \ldots ,0)$ must be and then show that it works.

3. Let $g: \mathbb{R} \to \mathbb{R}^2$ by $g(x) = (g_1(x), g_2(x))$. Prove that g is differentiable at $a \in \mathbb{R}$ if and only if $g_1(x)$ and $g_2(x)$ are and in that case $Dg(a) = \begin{pmatrix} g_1'(a) \\ g_2'(a) \end{pmatrix}$. Hint: For any point $(c_1, c_2) \in \mathbb{R}^2$,

$$|c_1| \le \sqrt{c_1^2 + c_2^2} = |(c_1, c_2)| \le |c_1| + |c_2|$$
$$|c_2| \le \sqrt{c_1^2 + c_2^2} = |(c_1, c_2)| \le |c_1| + |c_2|.$$

4. Let $f(x, y) = \frac{x^2 y}{x^4 + y^4}$ if $(x, y) \neq (0,0)$

$= 0$ if $(x, y) = (0,0)$.

Determine if $f(x, y)$ is differentiable at $(0,0)$.