# Orbits, Cycles, and the Alternating Groups

Find the orbits of the following permutations.

1. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 1 & 2 \end{pmatrix}$
2. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 5 & 4 & 3 \end{pmatrix}$.

Calculate the product of the cycles.

3. $(1,\ 3,\ 4,\ 6)(2,\ 5,\ 7)$
4. $(1,\ 3)(2,\ 6,\ 7)(4,\ 5,\ 8)$

Use for problems 5-8.

$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 5 & 4 & 3 \end{pmatrix}$; $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 6 & 1 & 5 & 3 & 2 \end{pmatrix}$.

5. Write $\sigma$ and $\tau$ as the product of disjoint cycles.
6. Write $\sigma$ and $\tau$ as the product of transpositions.
7. Calculate the order of $\sigma$ and $\tau$ in $S_6$.
8. Is $\sigma$ even or odd? Is $\tau$ even or odd?

Use for problems 9-12.

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 1 & 7 & 2 & 8 & 5 & 3 & 6 \end{pmatrix}.$$

9. Write $\sigma$ as the product of disjoint cycles.

10. Write $\sigma$ as the product of transpositions.

11. Calculate the order of $\sigma$ in $S_8$.

12. Is $\sigma$ even or odd?

13. $S_3 = \{\rho_0, \rho_1, \rho_2, \mu_1, \mu_2, \mu_3\}$ where

$$\rho_0 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \qquad \mu_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$$

$$\rho_1 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \qquad \mu_2 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$$

$$\rho_2 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix} \qquad \mu_3 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}.$$

Identify the elements of $A_3$.

14. For $n \geq 2$, let $\tau$ be an odd permutation in $S_n$. Show that every odd permutation $\mu$ in $S_n$ can be written as $\mu = \sigma\tau$ where $\sigma \in A_n$.

15. Let $G$ be a group and $x \in G$. Show that $\sigma_x: G \to G$ by $\sigma_x(g) = xg$ is a permutation of $G$.