Uniform Convergence of Series: The Weierstrass M-Test: HW Problems

For problems 1-6 determine where the series converges/diverges and where the series converges uniformly. In each case show why your answer is correct.

1. $\sum_{n=1}^{\infty} \left(\frac{x}{4}\right)^n$

2. $\sum_{n=1}^{\infty} ne^{-nx}$

3. $\sum_{n=1}^{\infty} \frac{1}{n(1+x^2)^n}$

4. $\sum_{n=1}^{\infty} \frac{\cos[(2n-1)x]}{(2n-1)^2}$

5. $\sum_{n=1}^{\infty} \frac{1}{n^4+x^4}$

6. $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$.

7. For what values of a does the series $\sum_{n=1}^{\infty} xn^a e^{-nx}$ converge uniformly on $x \geq 0$.

8. Suppose that $\sum_{n=1}^{\infty} |a_n| < \infty$ and $\sum_{n=1}^{\infty} |b_n| < \infty$. Prove that $\sum_{n=1}^{\infty} [a_n \cos(nx) + b_n \sin(nx)]$ converges uniformly on $\mathbb{R}$.