Approximating Lebesgue Measurable Sets- HW Problems

1. If E is a measurable set, show that $E + r$ and rE are measurable for any $r \in \mathbb{R}$.

2. If $m^*(E) = 0$ prove that there is a Borel set G, where $E \subseteq G$ and $m^*(G) = 0$.

3. Show that a set E is measurable if and only if for every $\epsilon > 0$ there is an open set O and a closed set F with $F \subseteq E \subseteq O$ and $m^*(O{\sim}F) < \epsilon$.

4. Suppose that $m^*(E) < \infty$. Show that if E is not measurable then there exists an open set $O \supseteq E$, with $m^*(O) < \infty$ such that

$$m^*(O{\sim}E) > m^*(O) - m^*(E).$$