

Open and Closed Sets in a Metric Space- HW Problems

1. Prove the following (assume the standard metric on \mathbb{R} and \mathbb{R}^2):
 - a. $(-2,2)$ is an open set in \mathbb{R} .
 - b. $[-2,2]$ is a closed set in \mathbb{R} .
 - c. $(-2,2]$ is neither an open set nor a closed set in \mathbb{R} .
 - d. Is $A = \{(x,y) \mid -2 < x < 2, y = 0\}$ open in \mathbb{R}^2 ? Prove your answer.

3. Let $A, B, C \subseteq X, d$ be non-empty open sets in a metric space X . Prove the following (without using the theorem that states that the union of open sets is open and the finite intersection of open sets is open).
 - a. $A \cup B \cup C$ is open in X .
 - b. $A \cap B \cap C$ is open in X .

4. Prove that If X, d is a metric space and $E \subseteq F \subseteq X$, then $\bar{E} \subseteq \bar{F}$.