Taylor Series- HW Problems

Find the Taylor series from the definition of a Taylor series for the following functions around the point given.

1. $f(x) = e^{-3x}; \qquad a = 0$

2. $g(x) = \sin(x); \qquad a = 0$

3. $f(x) = \sin(x); \qquad a = \dfrac{\pi}{2}$

4. $g(x) = \ln(x); \qquad a = 1$

5. Show that the Taylor series for $g(x) = \sin(x)$ in problem number 2 converges for all x.

Use known Maclaurin series to find a Maclaurin series for the following functions.

6. $f(x) = \cosh(x) = \dfrac{e^x + e^{-x}}{2}$

7. $g(x) = \dfrac{\cos(x) - 1 + \frac{1}{2}x^2}{x^3}$

8. $f(x) = x\ln(1 + x^2)$

9. $g(x) = \dfrac{x}{\sqrt{3+x}}$

10. Use a Maclaurin series to approximate the value of the following definite integrals so that the absolute value of the error is less than 10^{-4}.

a. $\int_0^1 \sin(x^3)\, dx$

b. $\int_0^1 x^2 e^{-x^2}\, dx$

Evaluate the following limits using a Maclaurin series.

11. $\lim\limits_{x\to 0} \dfrac{e^{(x^2)} - 1 - x^2 - \frac{1}{2}x^4}{x^6}$

12. $\lim\limits_{x\to 0} \dfrac{\ln(1+x) - x + \frac{1}{2}x^2 - \frac{1}{3}x^3}{x^4}$