Gradient, Divergence, and Curl- HW Problems

1. Find the divergence of the following vector fields.

$a.$ $\vec{V}(x,y,z) = x^2\vec{\imath} + y^2\vec{\jmath} + 2xz\vec{k}$

$b.$ $\vec{V}(x,y,z) = (\cos(xy))\vec{\imath} - (ysin(xy))\vec{\jmath}$.

2. Find the curl of the following vector fields.

$a.$ $\vec{V}(x,y,z) = x^2\vec{\imath} + y^2\vec{\jmath} + z^2\vec{k}$

$b.$ $\vec{V}(x,y,z) =< yz, 3xz, 2xy >$.

3. Find the scalar curl of the vector field $\vec{F}(x,y) = y^2\vec{\imath} + x^2\vec{\jmath}$.

4. Which of the following vector fields could be gradient fields? Which could be the curl of some vector field in $\mathbb{R}^3$?

$a.$ $\vec{F}(x,y,z) = y\vec{\imath} + (x+z)\vec{\jmath} + y\vec{k}$

$b.$ $\vec{F}(x,y,z) = (x-y)\vec{\imath} - (\sin(z))\vec{\jmath} + \vec{k}$

$c.$ $\vec{F}(x,y,z) = \big(ycos(z)\big)\vec{\imath} + (\mathrm{x}\cos(z))\vec{\jmath} - (xysin(z))\vec{k}$.

5. Let $\vec{V}(x,y,z) =< xz,\ y^2z,\ 2xy >$. Verify the $\nabla \cdot \left(\nabla \times \vec{V}\right) = 0$.

6. Let $f(x,y,z) = x^2y + y^2z$. Verify that $\nabla \times (\nabla f) = 0$.