Reparametrization of Curves and Closed Curves- HW Problems

1. Consider the curve $\gamma(t) = (2t,\ \frac{4\sqrt{2}}{3}t^{\frac{3}{2}},\ t^2)$ for $t > 0$.

a. Show that γ is a regular curve.

b. Find the arc length function $s(t)$ for γ starting at $t_0 = 0$.

For problems 2 and 3

a. Show that γ is a regular curve.

b. Find the arc length function $s(t)$ for γ starting at $t_0 = 0$

c. Find a unit speed reparametrization of γ (i.e., write γ as a function of s, its arc length).

2. Let $\gamma(t) = (\cosh(t), t),\ t \in \mathbb{R}$. Recall that

$\cosh(t) = \frac{(e^t+e^{-t})}{2},\ \sinh(t) = \frac{e^t-e^{-t}}{2},\ \cosh^2(t) - \sinh^2(t) = 1,$

$\frac{d}{dt}(\cosh(t)) = \sinh(t),$ and $\frac{d}{dt}(\sinh(t)) = \cosh(t).$

3. Let $\gamma(t) = (e^t\cos(t), e^t\sin(t), e^t),\ t \in \mathbb{R}$.

4. Show that the Limacon given by:

$$\gamma(t) = \big((1 + 2\cos(t))\cos(t)\,, (1 + 2\cos(t))\sin(t)\big); \quad t \in \mathbb{R}$$

is a closed curve with exactly one intersection point. (Hint: find the period, T, of γ and find points a, b where $\gamma(a) = \gamma(b)$ but $a - b$ in not an integer multiple of T).