Differentiation and Directional Derivatives- HW Problems

1. Let $f: \mathbb{R}^2 \to \mathbb{R}$ by $f(x,y) = \sqrt{|xy|}$. Show that f is not differentiable at $(0,0)$.

2. Let $g: \mathbb{R}^n \to \mathbb{R}$ where $|g(x)| \leq |x|^2$. Prove that g is differentiable at $(0,0,\dots,0)$. Hint: Figure out what $Dg(0,\dots,0)$ must be and then show that it works.

3. Let $g: \mathbb{R} \to \mathbb{R}^2$ by $g(x) = (g_1(x), g_2(x))$. Prove that g is differentiable at $a \in \mathbb{R}$ if and only if $g_1(x)$ and $g_2(x)$ are and in that case $Dg(a) = \begin{pmatrix} g_1'(a) \\ g_2(a) \end{pmatrix}$.

4. Let $f(x,y) = \frac{x^2 y}{x^4+y^4}$ if $(x,y) \neq (0,0)$

$= 0$ if $(x,y) = (0,0)$.

Determine if $f(x,y)$ is differentiable at $(0,0)$.

5. Find the directional derivative by calculating

$D_{\vec{u}}F(x) = \frac{d}{dt}\left(F(x + t\vec{u})\right)$ at $t = 0$.

Check your answer by calculating the directional derivative by $D_{\vec{u}}F(x) = (DF(x))\vec{u}$. (You can calculate $DF(x)$ by calculating the Jacobian matrix of partial derivatives)

a. $F(x, y) = (x^3 - y^3, xy)$ at $(x, y) = (2,3)$ in the direction $\vec{u} = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

b. $F(x, y, z) = (x + y + z, xy, yz)$ at $(x, y, z) = (1,2,3)$ in the direction $\vec{u} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{6}}\right)$.