Groups- HW Problems

In problems 1-7 determine if the operation $*$ defines a group structure on the set. If not, identify which group axioms are violated.

1. $a * b = ab$ on $\mathbb{Z}^+$
2. $a * b = a + b$ on $3\mathbb{Z} = \{3n | n \in \mathbb{Z}\}$
3. $a * b = ab$ on $\mathbb{R}$
4. $a * b = ab$ on $\mathbb{Q}^* = \{x \in \mathbb{Q} | x \neq 0\}$
5. $A * B = AB,$ matrix multiplication on

$$M = \{A \in M_n(\mathbb{R}) | A \text{ is diagonal}\}$$

6. $A * B = A + B,$ matrix addition on $M_n(\mathbb{R})$
7. $A * B = AB,$ matrix multiplication on

$$M = \{A \in M_n(\mathbb{R}) | \det(A) = \pm 1\}$$

8. Give a multiplication table for $\{0,1,2,3,4\}$ with $a * b = a + b$ (mod 5) where $a + b$ (mod 5) is the remainder when $a + b$ is divided by 5. Find the inverse of each element of $\{0,1,2,3,4\}$.

9. Suppose G is a group. Prove that G has exactly one element g such that $g * g = g$.

10. Suppose $(a * b) * (a * b) = (a * a) * (b * b)$ for all $a, b \in G$, where G is a group. Prove that $a * b = b * a$ (ie G is an abelian group).