The Gauss Equations and the Codazzi-Mainardi Equations-
HW Problems

1. Calculate the 8 Christoffel symbols and then calculate the Gauss
curvature by using the formula on page 8 of the class notes for

$$\vec{\Phi}(u,v) = (ucos(v),\ usin(v),\ u) \quad \text{(cone)}$$

2. Consider the two surfaces given by

$$\vec{\Phi}(u,v) = (vcos(u),\ vsin(u),\ u) \quad \text{(helicoid)}$$

$$\vec{\Psi}(u,v) = (vcos(u),\ vsin(u),\ \ln(v))\ .$$

Calculate the first and second fundamental forms for each surface

and then the Gauss curvature from them (by $K = \frac{\det(F_2)}{\det(F_1)}$). Show that

the two surfaces have the same Gaussian curvature but different

first fundamental forms. This shows the converse of Gauss' theorem

Egregium is not true: Two surfaces with the same first fundamental

form must have the same Gauss curvature, but having the same

Gauss curvature does not imply the first fundamental forms must be

the same.