

The Gamma Function and Bessel Functions- HW Problems

1. Use the fact that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ to find $\Gamma\left(\frac{5}{2}\right)$.
2. Show that $\Gamma(x + 5) = (x + 4)(x + 3)(x + 2)(x + 1)x\Gamma(x)$.
3. Find $J_3(x)$ in terms of $J_0(x)$ and $J_1(x)$.
4. Show that $y = xJ_1(x)$ is a solution to the differential equation $xy'' - y' - x^2J_0'(x) = 0$.

Hint: $J_1(x)$ satisfies Bessel's equation:

$$x^2J_1''(x) + xJ_1'(x) + (x^2 - 1)J_1(x) = 0,$$

and $J_0'(x) = -J_1(x)$.

5. Write $\int x^4J_0(x)dx$ in terms of Bessel functions and $\int J_0(x)dx$.