

Differentiability of Monotonic Functions- HW Problems

1. Find $\bar{D}f(0)$ and $\underline{D}f(0)$ for the functions:

a. $f(x) = |x|.$

b. $g(x) = x$ if $x \in \mathbb{Q}$
 $= 0$ if $x \notin \mathbb{Q}.$

c. $h(x) = x \sin\left(\frac{1}{x}\right)$ if $x \neq 0$
 $= 0$ if $x = 0.$

2. Suppose that f is integrable over $[a, b]$. We say that the antiderivative of f is a function g on $[a, b]$ such that

$$g(x) = \int_a^x f; \quad x \in [a, b].$$

Show that g is differentiable a.e. on $[a, b]$.

3. Let f be a continuous function on \mathbb{R} . Must there be an open interval on which f is monotone? Explain.

Hint: There exists a function on \mathbb{R} which is continuous everywhere but differentiable nowhere.

4. Suppose that f is a function on (a, b) and $a < c < b$ is a local minimum of f (ie there is a neighborhood, N , around $x = c$ such that $f(c) \leq f(x)$ for $x \in N$). Show that $\underline{D}f(c) \leq 0 \leq \bar{D}f(c)$.