Taylor Series- HW Problems

1. Use the Taylor polynomial $T_4(x)$ around a=0 to approximate cos(0.1). How large could the error be?

2. Approximate $e^{-0.1}$ to within an error of 0.00001.

3. Find the values of x where $T_4(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}; \quad x > 0,$ has an error of less than 0.01 for the function $f(x) = \ln(1 + x)$.

4. Prove that the Taylor series around $a = \frac{\pi}{2}$ for $f(x) = cosx$ converges to $f(x)$ for all $x\epsilon\mathbb{R}$.

5. Prove that the Taylor series around $a = 0$ for $f(x) = e^{-2x}$ converges to $f(x)$ for all $x\epsilon\mathbb{R}$.

6. Let $f(x) = \ln(1+x)$. Below are some derivatives of $f(x)$ which you can use to answer parts a-d.

$f'(x) = \frac{1}{1+x}$

$f''(x) = -\frac{1}{(1+x)^2}$

$f'''(x) = \frac{2}{(1+x)^3}$

$f''''(x) = -\frac{6}{(1+x)^4}$

$f^{(n+1)}(x) = \frac{(-1)^n(n!)}{(1+x)^{n+1}}$

a. Find the 3rd Taylor polynomial, $T_3(x)$, around $a = 0$ for $f(x)$.

b. Approximate $\ln(1.1)$ using $T_3(x)$, around $a = 0$ (You can leave your answer as a sum of "messy" fractions).

c. Find a bound for the error in this approximation.

d. Prove that the Taylor series for $f(x)$, which is $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x)^n}{n}$ (you don't need to find this formula), converges to $\ln(1+x)$ for all $0 < x < 1$.