Rings of Polynomials- HW Problems

1. Let $f(x) = x^2 + 3x + 2$ and $g(x) = x^2 + 2x + 4$ in $\mathbb{Z}_5[x]$. Find $f(x) + g(x)$ and $[f(x)][g(x)]$.

2. Let $f(x) = 4x^4 + 3x^2 + 6$ and $g(x) = 2x^3 + 3x + 2$ in $\mathbb{Z}_8[x]$. Find $f(x) + g(x)$ and $[f(x)][g(x)]$.

3. How many distinct polynomials of degree less than or equal to 2 are there in $\mathbb{Z}_3[x]$?

4. Let ϕ_k be the evaluation homomorphism at $x = k$. Evaluate

$$\phi_3[(x^4 + 3)(2x^2 + 4)(x^6 - 2x^3 + 1)] \text{ in } \mathbb{Z}_5[x].$$

In problems 5-7 find all of the zeros of the polynomials in the given ring.

5. $x^2 + x + 1$ in $\mathbb{Z}_3[x]$.
6. $x^4 + 2x^3 + x^2 + 1$ in $\mathbb{Z}_5[x]$.
7. $x^3 + 2x^2 + 3x + 2$ in $\mathbb{Z}_4[x]$.

8. Find a polynomial of positive degree that is a unit in $\mathbb{Z}_4[x]$.

9. Identify the units in $\mathbb{Z}_7[x]$ and the units in $\mathbb{Z}[x]$.