The Divergence Theorem- HW Problems

Verify the divergence theorem, $\iint_{\partial W} \vec{F} \cdot d\vec{S} = \iiint_W Div\, \vec{F} dV$, for

1. $\vec{F}(x, y, z) = (z)\vec{\imath} + (y)\vec{\jmath} + (x)\vec{k}$, where W is the solid ball $x^2 + y^2 + z^2 \leq 1$.

Use the divergence theorem to solve problems 2-9.

2. Evaluate $\iint_S \vec{F} \cdot d\vec{S}$ where $\vec{F}(x, y, z) = (x^2)\vec{\imath} - (y)\vec{\jmath} + (z)\vec{k}$, and $S = \partial W$ where W is the solid cylinder $x^2 + y^2 \leq 4, \quad 0 \leq z \leq 3$.

3. Calculate the flux of
$$\vec{F}(x, y, z) = (x + yz)\vec{\imath} + (y + xz)\vec{\jmath} + (z + xy)\vec{k}$$
out of the sphere $x^2 + y^2 + z^2 = 4$.

4. Evaluate $\iint_S \vec{F} \cdot d\vec{S}$ where
$$\vec{F}(x, y, z) = (x^3 + y^3)\vec{\imath} + (y^3 + z^3)\vec{\jmath} + (z^3 + x^3)\vec{k}$$
and S is the unit sphere (outward normal)

5. Calculate the flux of
$$\vec{F}(x, y, z) = (y^3 + e^y)\vec{\imath} + (z + x^3)\vec{\jmath} + (z^3)\vec{k}$$
where W is the region in $\mathbb{R}^3$ given by $x^2 + y^2 + z^2 \leq 4,\ z \leq 0,$ $y \leq 0$

6. Evaluate $\iint_S \vec{F} \cdot d\vec{S}$ where

$$\vec{F}(x,y,z) = (x+y)\vec{\imath} + (y+z)\vec{\jmath} + (z+e^{\cos(x)})\vec{k}$$

and W is the region in $\mathbb{R}^3$ given by $x^2+y^2 \leq z \leq 8-x^2-y^2$, $x \leq 0$, where $S = \partial W$ is oriented with the outward pointing normal.

7. Evaluate $\iint_{\partial W} \vec{F} \cdot d\vec{S}$ where

$$\vec{F}(x,y,z) = (x+e^z)\vec{\imath} + (y-z)\vec{\jmath} + (z)\vec{k}$$

and W is the region in $\mathbb{R}^3$ given by the intersection of $z \leq 6-x^2-y^2$ and $z \geq 2$, where ∂W is oriented with the outward pointing normal.

8. Calculate the flux of

$$\vec{F}(x,y,z) = < x+yz,\ y-xz,\ x^2+y^2 >$$

out of the rectangular solid $[0,1] \times [1,3] \times [2,4]$.

9. Let S be a closed surface in $\mathbb{R}^3$ and $\vec{F}(x,y,z)$ a C^2 vector field on $\mathbb{R}^3$. Evaluate $\iint_S (\nabla \times \vec{F}) \cdot d\vec{S}$.