The Curvature Tensor- HW Problems

1. Let the unit sphere in $\mathbb{R}^3$ be parametrized by

   $\overrightarrow{\Phi}(\theta,\phi) = (cos\theta sin\phi, sin\theta sin\phi, cos\phi), \quad (\theta,\phi) \in (0,2\pi) \times (0,\pi),$

   with the induced metric

   $$g = \begin{pmatrix} \sin^2\phi & 0 \\ 0 & 1 \end{pmatrix}.$$

   Using the Christoffel symbols calculated in a previous assignment:

   a. Find the components of the Riemann curvature tensor, R^i_{jkl}, for g.

   b. Find the components of the Ricci tensor, R_{ij}, and the scalar curvature, R.

   c. Show that g is an Einstein metric.

2. Let $H = R^2_+ = \{(x,y) \in \mathbb{R}^2 | \, y > 0\}$ with the metric

   $$g = \frac{1}{y^2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

   Using the Christoffel symbols calculated in a previous assignment:

   a. Find the components of the Riemann curvature tensor, R^i_{jkl}, for g.

   b. Find the components of the Ricci tensor, R_{ij}, and the scalar curvature, R.

   c. Show that g is an Einstein metric.