

The General Lebesgue Integral- HW Problems

1. In each case below show that $\lim_{n \rightarrow \infty} \int_E f_n = 0$, where $E = [0,1]$.

a. $f_n(x) = \frac{nx}{1+n^2x^2}$

b. $f_n(x) = \frac{n\sqrt{x}}{1+n^2x^2}$ Hint: $1 + n^2x^2 \geq 2nx$.

2. Evaluate $\lim_{n \rightarrow \infty} \int_E \frac{e^{-\frac{x^2}{n}}}{x^2}$ where $E = [2, \infty)$. Make sure you justify the steps in your calculation.

3. Evaluate $\lim_{n \rightarrow \infty} \int_E \frac{e^{\frac{\sin x}{n}}}{x^2}$ where $E = [0, \infty)$. Make sure you justify the steps in your calculation.

4. Let $g \geq 0$, a measurable function on \mathbb{R} . Show that

$$\lim_{n \rightarrow \infty} \int_{-n}^n g = \int_{\mathbb{R}} g.$$