

The First Fundamental Form: Surface Area- HW Problems

1. Find the surface area of the portion of the cone given by

$$\vec{\Phi}(u, v) = (au(\cos(v)), au(\sin(v)), u); \quad a > 0, \quad 0 \leq u \leq b, \quad 0 \leq v \leq 2\pi.$$

2. Suppose a surface is given as $z = f(x, y)$ and you want to find the surface area that lies over a region D in the xy plane. Let

$$\vec{\Phi}(u, v) = (u, v, f(u, v)) \text{ represent this surface.}$$

Calculate the first fundamental form of $\vec{\Phi}$ and show that the surface area is

$$\text{surface area} = \iint_D \sqrt{1 + (f_x)^2 + (f_y)^2} \, dx dy.$$

3. Find the surface area of the hyperbolic paraboloid $z = y^2 - x^2$ that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$ (you can use the result in #2).

4. Find the surface area of the surface given by

$$\vec{\Phi}(u, v) = (u, v, uv); \quad u^2 + v^2 < 9.$$