

Elementary Matrices- HW Problems

1. Identify which of the following matrices are elementary matrices.

a. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ b. $\begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix}$ c. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}$ d. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2. Let

$$A = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 2 & 3 \\ 2 & -2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -2 & 1 \\ -1 & -4 & 3 \\ 2 & -4 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & -2 & 1 \\ 3 & -12 & 5 \\ 2 & -4 & 1 \end{bmatrix}.$$

Find elementary row/column operations that turn A into B and turn B into C . Find elementary matrices E and F such that $AE = B$ and $FB = C$.

3. Let

$$A = \begin{bmatrix} 1 & 1 & 0 \\ -2 & 1 & 3 \\ 4 & 2 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 & 0 \\ 6 & 5 & 1 \\ 4 & 2 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} -5 & -4 & -1 \\ 6 & 5 & 1 \\ 4 & 2 & -1 \end{bmatrix}.$$

Find elementary matrices E and F such that $EA = B$ and $FB = C$.

4. Find the inverse matrix of the the following elementary matrices.

a. $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ c. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$

5. Let $A = \begin{bmatrix} 1 & -2 & 2 \\ 2 & -2 & 3 \\ 2 & -3 & 3 \end{bmatrix}$. Find a sequence of elementary row/column

operations that transforms A into the identity matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.