

Factor/Quotient Groups- HW Problems

For problems 1-3 find the order of the factor group.

1. $(\mathbb{Z}_5 \times \mathbb{Z}_3)/(\{0\} \times \mathbb{Z}_3)$
2. $(\mathbb{Z}_8 \times \mathbb{Z}_{20})/\langle (4,5) \rangle$
3. $(\mathbb{Z}_{13} \times \mathbb{Z}_{15})/\langle (1,1) \rangle$

- 4a. What is the order of $(15,6)$ in $\mathbb{Z}_{40} \times \mathbb{Z}_{20}$?
- b. Find the order of $(\mathbb{Z}_{40} \times \mathbb{Z}_{20})/\langle (15,6) \rangle$.

- 5a. What is the order of $(25,6)$ in $\mathbb{Z}_{30} \times \mathbb{Z}_{20}$?
- b. Find the order of $(\mathbb{Z}_{30} \times \mathbb{Z}_{20})/\langle (25,6) \rangle$.

6. $\phi: GL(n, \mathbb{R}) \rightarrow \mathbb{R}^*$ by $\phi(A) = \det(A)$ is a homomorphism.
 - a. Find the kernel of ϕ .
 - b. What is the isomorphism one gets from this homomorphism based on the fundamental homomorphism theorem for groups?

For problems 7-9 find the order of the element in each factor group.

7. $34 + \langle 8 \rangle$ in $\mathbb{Z}_{40}/\langle 8 \rangle$
8. $(3,2) + \langle (1,1) \rangle$ in $(\mathbb{Z}_4 \times \mathbb{Z}_4)/\langle (1,1) \rangle$
9. $(5,2) + \langle (4,1) \rangle$ in $(\mathbb{Z}_{12} \times \mathbb{Z}_3)/\langle (4,1) \rangle$.

10. Write down the distinct elements of the factor group $\mathbb{Z}_{12}/\langle 3 \rangle$.

11. Show that A_n is a normal subgroup of S_n by finding a homomorphism on S_n for which A_n is its kernel.

12. Let H_1 and H_2 be normal subgroups of a group G . Show that

$$H_1 \cap H_2 = \{g \in G \mid g \in H_1, g \in H_2\}$$

is also a normal subgroup of G .