Linear Approximation and Differentials- HW Problems

1. Use the linear approximation of $f(x) = \sqrt[3]{x}$ at $a = 8$ to approximate $\sqrt[3]{8.1}$ and $\sqrt[3]{7.9}$.

2. Show that the linear approximation of $f(x) = \tan(x)$ at $a = 0$ is $L(x) = x$.

3. Use linear approximations to approximate:

a. $(2.001)^4$

b. $(8.04)^{\frac{2}{3}}$

4. Find the differential dy of each of the following functions.

a. $y = \sec(\sqrt{x})$

b. $y = \dfrac{x}{1+x}$

5. The radius of a circular disk is measure as 6 in. with a maximum error of 0.3 in.

a. Estimate the maximum error in the calculated area of the disk.

b. Estimate the maximum error in the calculated circumference of the disk.

c. What is the maximum percentage error in the area of the disk?

6. The surface area of a sphere ($SA = 4\pi R^2$) is measured as $500 \; in^2$ with a maximum error of $2 \; in^2$.

a. Estimate the maximum error in the calculated volume (Hint: one way to do this is to find a formula for the volume of a sphere in terms of its surface area.).

b. What is the maximum percentage error?